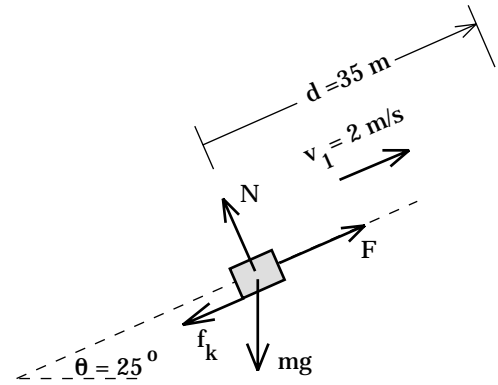


CHAPTER 6 -- ENERGY

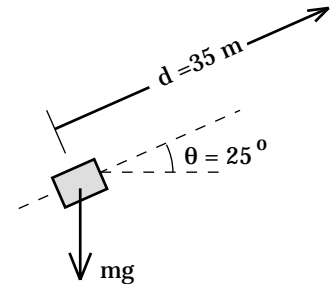
6.1) The f.b.d. shown to the right has been provided to identify all the forces acting on the body as it moves up the incline.

a.) To determine the work done by gravity as the body moves up the incline, there are two approaches. For your convenience, the force, velocity, and displacement are pictured below and also to the right.



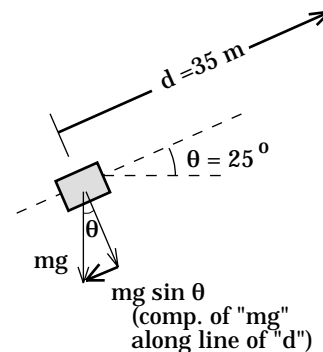
Approach #1: Using the definition of work and the angle between mg and d as ϕ :

$$\begin{aligned} W_{\text{grav}} &= \mathbf{F}_{\text{grav}} \cdot \mathbf{d} \\ &= (mg) (d) \cos \phi \\ &= (mg) (d) \cos (25^\circ + 90^\circ) \\ &= (3 \text{ kg})(9.8 \text{ m/s}^2)(35 \text{ m}) (-.423) \\ &= -434.87 \text{ joules.} \end{aligned}$$



Approach #2: Using the component of mg along the line of d :

$$\begin{aligned} W_{\text{grav}} &= \mathbf{F}_{\text{grav}} \cdot \mathbf{d} \\ &= \pm(F_{\text{mg parallel to "d"}}) (d) \\ &= -(mg \sin 25^\circ) (d) \\ &= -[(3 \text{ kg})(9.8 \text{ m/s}^2)(.423)] (35 \text{ m}) \\ &= -434.87 \text{ joules.} \end{aligned}$$

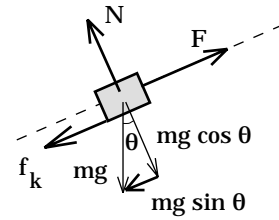


Note: In this case, the force is OPPOSITE the direction of the displacement which means the work must be negative. The *negative sign* in this case must be inserted *manually*. An alternative would be to notice that the angle ϕ between d and F 's component along d 's line is 180° and determine the work quantity using:

$$\begin{aligned}
 W_{\text{grav}} &= \mathbf{F}_{\text{grav}} \cdot \mathbf{d} \\
 &= (F_{\text{mg parallel to "d"}}) (d) \cos \phi \\
 &= (mg \sin 25^\circ) (d) \cos 180^\circ.
 \end{aligned}$$

In doing so, the cosine function will give you the -1 automatically.

b.) The frictional force is equal to $\mu_k N$. To determine N , we need to use an f.b.d. and N.S.L. in the *normal direction*. The f.b.d. is shown to the right. N.S.L. yields:



$$\begin{aligned}
 \underline{\Sigma F_N}: \quad N - mg \cos \theta &= 0 \quad (\text{as } a_N = 0) \\
 \Rightarrow N &= mg \cos \theta \\
 \Rightarrow f_k &= \mu_k N \\
 &= \mu_k (mg \cos \theta) \\
 &= (.3) (3 \text{ kg})(9.8 \text{ m/s}^2) \cos 25^\circ \\
 &= 7.99 \text{ nts.}
 \end{aligned}$$

Friction is always opposite the direction of motion. The *work* friction does will be:

$$\begin{aligned}
 W_f &= \mathbf{f}_k \cdot \mathbf{d} \\
 &= (f_k) (d) \cos 180^\circ \\
 &= -f_k d \\
 &= -(7.99 \text{ nts})(35 \text{ m}) \\
 &= -279.65 \text{ joules.}
 \end{aligned}$$

c.) The angle between d and N is 90° . The cosine of 90° is zero. That means that the *net work* done by the *normal force* will be zero . . . ALWAYS!

d.) Kinetic energy is defined as $(1/2)mv^2$. Using that expression we get:

$$\begin{aligned}
 KE_1 &= (1/2)mv^2 \\
 &= .5 (3 \text{ kg}) (2 \text{ m/s})^2 \\
 &= 6 \text{ joules.}
 \end{aligned}$$

e.) The work/energy theorem states:

$$W_{\text{net}} = \Delta \text{KE}.$$

For this case, that means:

$$\begin{aligned} W_{\text{net}} &= \Delta \text{KE}. \\ W_f + W_F + W_{\text{mg}} &= \text{KE}_2 - \text{KE}_1. \\ (f_k)(d) \cos 180^\circ + F(d) \cos 0^\circ + (mg)(d) \cos \phi &= (1/2)mv_2^2 - (1/2)mv_1^2. \end{aligned}$$

Plugging in the numbers, we get:

$$\begin{aligned} (-279.65 \text{ J}) + F(35 \text{ m}) + (-434.87 \text{ J}) &= (1/2)(3 \text{ kg})(7 \text{ m/s})^2 - (1/2)(3 \text{ kg})(2 \text{ m/s})^2 \\ \Rightarrow F &= 22.34 \text{ nts}. \end{aligned}$$

6.2) The situation is shown in the sketch to the right. We need to derive a *general algebraic expression* for the force F acting on the block, given the fact that that force is always oriented at an angle of 12° relative to the direction of motion.

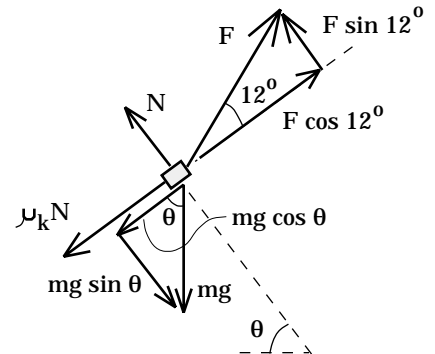
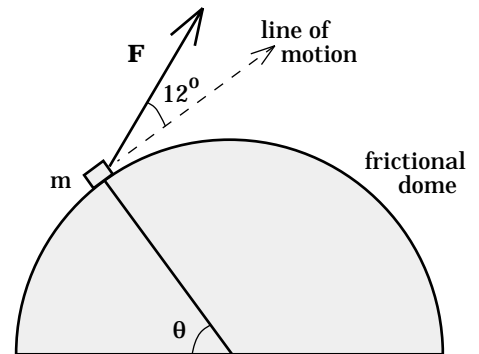
Consider the f.b.d. (shown below) for the forces acting on the body when located at an arbitrary angle. As v is small and constant, both the centripetal acceleration (i.e., v^2/R) and translational acceleration (i.e., dv/dt) are zero or approximately zero. Therefore, N.S.L. and the f.b.d. yields:

$$\begin{aligned} \underline{\Sigma F_x}: \\ N - mg \sin \theta + F \sin 12^\circ &= 0 \quad (\text{as } a_N = 0) \\ \Rightarrow N &= (mg \sin \theta) - F \sin 12^\circ. \end{aligned}$$

Knowing the normal force, the frictional force follows as:

$$\begin{aligned} f_k &= \mu_k N \\ &= \mu_k (mg \sin \theta - F \sin 12^\circ). \end{aligned}$$

positioning of force F
when mass is at
an arbitrary angle θ



$$\underline{\Sigma F_{\text{tang}}}:$$

$$-\mu_k N - mg \cos \theta + F \cos 12^\circ = 0$$

$$\Rightarrow -\mu_k (mg \sin \theta - F \sin 12^\circ) - mg \cos \theta + F \cos 12^\circ = 0$$

$$\Rightarrow F = [\mu_k mg \sin \theta + mg \cos \theta] / [\mu_k \sin 12^\circ + \cos 12^\circ].$$

Rewriting this with the constants in front of the variable expression, we get:

$$F = \left[\frac{mg}{\cos 12^\circ + \mu_k \sin 12^\circ} \right] [\mu_k \sin \theta + \cos \theta].$$

Knowing the force in general terms, we can use $\int \mathbf{F} \cdot d\mathbf{r}$ to determine the work the force does as the body rises from $\theta = 20^\circ$ to $\theta = 60^\circ$. Noting that:

--The angle ϕ between the *line of F* and the *line of dr* is always 12° ;

--The magnitude of a differential displacement $d\mathbf{r}$ along an arc equals $R d\theta$, where R is the radius of the arc and $d\theta$ is the differential angle through which the motion occurs;

--And $\mu_k = .2$, $m = .5 \text{ kg}$, and $R = .3 \text{ meters}$, we can write:

$$W = \int \mathbf{F} \cdot d\mathbf{r}$$

$$= \int |\mathbf{F}| |d\mathbf{r}| \cos \phi$$

$$= \left[\frac{mg}{\cos 12^\circ + \mu_k \sin 12^\circ} \right] \int_{\theta=20^\circ}^{60^\circ} [\mu_k \sin \theta + \cos \theta] [R d\theta] \cos 12^\circ$$

$$= \left[\frac{mgR \cos 12^\circ}{\cos 12^\circ + \mu_k \sin 12^\circ} \right] \left[\int_{\theta=20^\circ}^{60^\circ} [\mu_k \sin \theta] d\theta + \int_{\theta=20^\circ}^{60^\circ} [\cos \theta] d\theta \right]$$

$$= \left[\frac{mgR \cos 12^\circ}{\cos 12^\circ + \mu_k \sin 12^\circ} \right] \left[[\mu_k (-\cos \theta)]_{\theta=20^\circ}^{60^\circ} + [(\sin \theta)]_{\theta=20^\circ}^{60^\circ} \right]$$

$$= \left[\frac{(.5 \text{ kg})(9.8 \text{ m/s}^2)(.3) \cos 12^\circ}{\cos 12^\circ + (.2) \sin 12^\circ} \right] \left[[(.)(-[\cos 60^\circ - \cos 20^\circ])] + [(\sin 60^\circ - \sin 20^\circ)] \right]$$

$$= .86 \text{ joules.}$$

6.3) All the energy is stored in spring potential energy. Using the potential energy function for a spring we get:

$$\begin{aligned} U_{\text{sp}} &= (1/2)kx^2 \\ &= .5(120 \text{ nt/m})(.2 \text{ m})^2 \\ &= 2.4 \text{ joules.} \end{aligned}$$

6.4) The relationship between the field's *potential energy* function (assumed known) and its associated *force* function is:

$$\begin{aligned} \mathbf{F}(\mathbf{x}, y, z) &= -\vec{\nabla} U(\mathbf{x}, y, z) \\ &= -\left(\frac{\partial(U)}{\partial x} \mathbf{i} + \frac{\partial(U)}{\partial y} \mathbf{j} + \frac{\partial(U)}{\partial z} \mathbf{k} \right) \\ &= -\left(\frac{\partial\left(-\frac{\mathbf{k}_1}{x} e^{-ky}\right)}{\partial x} \mathbf{i} + \frac{\partial\left(-\frac{\mathbf{k}_1}{x} e^{-ky}\right)}{\partial y} \mathbf{j} + \frac{\partial\left(-\frac{\mathbf{k}_1}{x} e^{-ky}\right)}{\partial z} \mathbf{k} \right) \\ &= -\left[\left(\frac{\mathbf{k}_1}{x^2}\right) e^{-ky} \mathbf{i} + \frac{\mathbf{k}_1}{x} (k e^{-ky}) \mathbf{j} \right] \end{aligned}$$

6.5) Looking at the function, the force will equal zero when $y = 0$ and when $\ln x = 3$ (i.e., when $x = 20.086$). Using this and our force/potential energy relationship, we get:

$$U(\mathbf{x}, y) - U(\mathbf{x}=20.086, y=0) = -\int \mathbf{F} \cdot d\mathbf{r}.$$

Observing that $U(\mathbf{x} = 20.086, y = 0) = 0$ and that $d\mathbf{r} = dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}$, and realizing that if this was a test question, 90% of the points would be wrapped up in the *first two lines* of what follows (i.e., the layout), we can write this as:

$$\begin{aligned} U(\mathbf{x}, y) &= -\int_{\mathbf{x}=20.086, y=0}^{\mathbf{x}, y} \left[(\mathbf{k}_1 \ln(x) - 3)\mathbf{i} - (\mathbf{k}_2 y^2)\mathbf{j} \right] \bullet [dx\mathbf{i} + dy\mathbf{j} + dz\mathbf{k}] \\ &= -\int_{\mathbf{x}=20.086, y=0}^{\mathbf{x}, y} \left[(\mathbf{k}_1 \ln(x) - 3)dx - (\mathbf{k}_2 y^2)dy \right] \\ &= -\int_{\mathbf{x}=20.086}^{\mathbf{x}} [\mathbf{k}_1 \ln(x) - 3]dx + \int_{y=0}^y [\mathbf{k}_2 y^2]dy \\ &= -\int_{\mathbf{x}=20.086}^{\mathbf{x}} [\mathbf{k}_1 \ln(x)]dx + \int_{\mathbf{x}=20.086}^{\mathbf{x}} [3]dx + \int_{y=0}^y [\mathbf{k}_2 y^2]dy. \end{aligned}$$

Setting the k terms equal to *one* and noting that $\int(\ln x)dx = (x)\ln x - x$, we can evaluate these integrals as:

$$\begin{aligned} U(\mathbf{x}, \mathbf{y}) &= -\left[\mathbf{x} \ln \mathbf{x} - \mathbf{x}\right]_{\mathbf{x}=20.086}^{\mathbf{x}} - \left[-3\mathbf{x}\right]_{\mathbf{x}=20.086}^{\mathbf{x}} + \left[\frac{\mathbf{y}^3}{3}\right]_{\mathbf{y}=0}^{\mathbf{y}} \\ &= -\left[(\mathbf{x} \ln \mathbf{x} - \mathbf{x}) - (20.086 \ln 20.086 - 20.086)\right] - \left[(-3\mathbf{x}) - (-3(20.086))\right] + \left[\left(\frac{\mathbf{y}^3}{3}\right) - \left(\frac{(0)^3}{3}\right)\right] \\ &= -\mathbf{x} \ln \mathbf{x} + 4\mathbf{x} + \frac{\mathbf{y}^3}{3} - 20.086. \end{aligned}$$

Note: A perfectly legitimate follow-up question to this problem might be: How much *work* does the force field do as the body goes from $x = 1, y = 2$ to $x = 4, y = -1$. The answer is:

$$\begin{aligned} W_{\text{cons.forc}} &= -\Delta U \\ &= -\left[U_{\text{pt2}} - U_{\text{pt1}} \right] \\ &= -\left[[-4\ln 4 + 4(4) + (-1)^3/3 - 20.09] - [-\ln(1) + 4(1) + (2)^3/3 - 20.09]\right] \\ &= -3.45 \text{ joules.} \end{aligned}$$

6.6)

a.) We could use the *work/energy theorem* on this problem, but the *modified conservation of energy equation* is so much easier to use that we will use it here. Noting that the tension in the line is always *perpendicular to the motion* (i.e., the work done due to tension is zero), we can write:

$$\begin{aligned} KE_1 + \sum U_1 + \sum W_{\text{extra}} &= KE_2 + \sum U_2 \\ (1/2)mv_1^2 + mgy_1 + Td\cos 90^\circ &= (1/2)mv_2^2 + mgy_2 \\ (0) + m(9.8 \text{ m/s}^2)(12 \text{ m}) + (0) &= (1/2)mv_2^2 + m(9.8 \text{ m/s}^2)(5 \text{ m}). \end{aligned}$$

Being careful not to confuse mass terms denoted by m and the units of length (meters, abbreviated m), we can cancel the mass terms and get:

$$v_2 = 11.71 \text{ m/s.}$$

b.) At the bottom of the arc, Tarzan's velocity can again be found using the *modified conservation of energy* expression (we need that velocity

because he is moving under the influence of a center-seeking force--a velocity driven function--at that point). Using the approach:

$$\begin{aligned} KE_1 + \Sigma U_1 + \Sigma W_{\text{extra}} &= KE_{\text{bot}} + \Sigma U_{\text{bot}} \\ (1/2)mv_1^2 + mgy_1 + (0) &= (1/2)mv_{\text{bot}}^2 + mgy_2 \\ (0) + m(9.8 \text{ m/s}^2)(12 \text{ m}) + (0) &= (1/2)mv_{\text{bot}}^2 + m(9.8 \text{ m/s}^2)(2 \text{ m}). \end{aligned}$$

Canceling the mass terms yields:

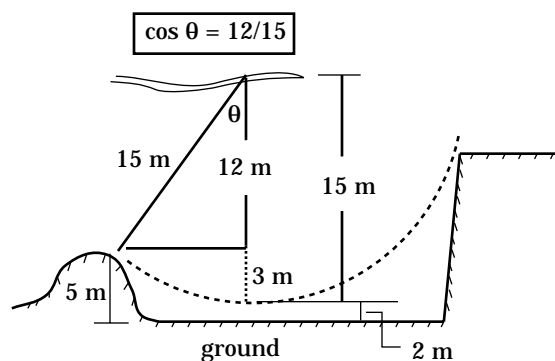
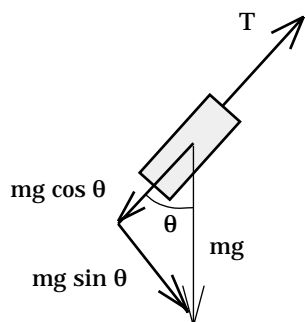
$$v_{\text{bot}} = 14 \text{ m/s.}$$

There is a *tension force* upward and *gravity* downward when Tarzan is at the bottom of the arc (you might want to draw an f.b.d. to be complete). N.S.L. implies:

$$\underline{\Sigma F_c}:$$

$$\begin{aligned} T - mg &= ma_c \\ &= m(v^2/R) \quad (\text{as } a_c \text{ is a centripetal acc.}) \\ \Rightarrow T &= mg + mv^2/R \\ &= (80 \text{ kg})(9.8 \text{ m/s}^2) + (80 \text{ kg})(14 \text{ m/s})^2/(15 \text{ m}) \\ &= 1829.3 \text{ nts.} \end{aligned}$$

c.) At the molehill, Tarzan's velocity is 11.71 m/s. An f.b.d. for that situation is shown below-left. The only thing that is really tricky about the problem is determining the angle θ . The diagram below-right will do that.



With θ , N.S.L. yields:

$$\underline{\Sigma F_c}:$$

$$\begin{aligned} T - mg \cos \theta &= ma_c \\ &= m(v^2/R). \end{aligned}$$

As a_c is a centripetal acceleration:

$$\begin{aligned} \Rightarrow T &= mg \cos \theta + mv^2/R \\ &= (80 \text{ kg})(9.8 \text{ m/s}^2)(12/15) + (80 \text{ kg})(11.71 \text{ m/s})^2/(15 \text{ m}) \\ &= 1358.5 \text{ nts.} \end{aligned}$$

6.7) Both gravity and friction do *work* as the body slides down the incline. Using the *modified conservation of energy equation*, we get:

$$\begin{aligned} KE_1 + \Sigma U_1 + \Sigma W_{\text{extra}} &= KE_2 + \Sigma U_2 \\ (1/2)mv_1^2 + mgy_1 + (-f_k d) &= (1/2)mv_2^2 + mgy_2 \\ (0) + mgR - (f_k d) &= (0) + (0) \\ \Rightarrow f_k &= mgR/d. \end{aligned}$$

In this case, d is the total distance over which the frictional force acts. That is, the 18 meters along the horizontal surface AND the quarter circumference down the curved incline (that will equal $(1/4)(2\pi R)$). That total distance is:

$$\begin{aligned} d &= 18 + .5\pi R \\ &= 18 + .5\pi(2 \text{ m}) \\ &= 21.14 \text{ m.} \end{aligned}$$

Plugging this into our expression, we get:

$$\begin{aligned} f_k &= mgR/d \\ &= (12 \text{ kg})(9.8 \text{ m/s}^2)(2 \text{ m})/(21.14) \\ &= 11.13 \text{ nts.} \end{aligned}$$

6.8) The fact that the angle is 85° makes no difference, assuming the velocity is great enough to allow the dart to make it to the monkey. What is important is that the dart has enough energy in the beginning to effect a pierce at the end. Using *conservation of energy*:

$$\begin{aligned} \text{KE}_1 + \text{SU}_1 + \text{SW}_{\text{ext}} &= \text{KE}_2 + \text{SU}_2 \\ (1/2)mv_1^2 + (0) + (0) &= (1/2)mv_2^2 + mgh_2. \end{aligned}$$

Dividing out the masses and multiplying by 2 yields:

$$\begin{aligned} v_1 &= [v_2^2 + 2gh_2]^{1/2} \\ &= [(4 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(35 \text{ m})]^{1/2} \\ &= 26.5 \text{ m/s}. \end{aligned}$$

6.9) Assuming an average frictional force of 27 newtons:

a.) Let the *potential energy equals zero* level be the ground. That means that below-ground level, the h in mgh will be *negative*. Using *conservation of energy*:

$$\begin{aligned} \text{KE}_1 + \Sigma U_1 + \Sigma W_{\text{ext}} &= \text{KE}_2 + \Sigma U_2 \\ (1/2)mv_1^2 + (0) + (-f_k d) &= (1/2)mv_C^2 + mgh_C. \end{aligned}$$

or

$$\begin{aligned} .5(1800 \text{ kg})(38 \text{ m/s})^2 - (27 \text{ nts})(130 \text{ m}) &= .5(1800 \text{ kg})v_C^2 + (1800 \text{ kg})(9.8 \text{ m/s}^2)(-15 \text{ m}) \\ \Rightarrow v_C &= 41.64 \text{ m/s}. \end{aligned}$$

b.) The only thing that is tricky about this is finding the total distance d traveled (we need that to determine the amount of work friction does). Noting that the distance traveled while moving through the loop is $2\pi R$, *conservation of energy* yields:

$$\begin{aligned} \text{KE}_1 + \Sigma U_1 + \Sigma W_{\text{ext}} &= \text{KE}_2 + \Sigma U_2 \\ (1/2)mv_1^2 + (0) + (-f_k d_{\text{tot}}) &= (0) + mgh_{\text{ramp}} \\ (1/2)mv_1^2 + 0 - f_k(70+60+40+2\pi(20)+d) &= 0 + mgd \sin \theta \\ .5(1800 \text{ kg})(38 \text{ m/s})^2 - [(27 \text{ nts})(295.7 \text{ m}) + 27d] &= (1800)(9.8 \text{ m/s}^2)d \sin 30^\circ \\ \Rightarrow d &= 146 \text{ m}. \end{aligned}$$

c.) Along with the *conservation of energy*, this problem requires N.S.L. We know that at the top of the arc, the *vertical forces* are centripetal at the

velocity required AND the *normal force* goes to zero. From the *conservation of energy* we get:

$$\begin{aligned}
 KE_1 + \Sigma U_1 + \Sigma W_{\text{ext}} &= KE_2 + \Sigma U_2 \\
 (1/2)mv_1^2 + (0) + (-f_k d_{\text{top}}) &= (1/2)mv_{\text{top}}^2 + mgh_{\text{top}} \\
 (1/2)mv_1^2 + 0 - f_k[70+60+40+(2\pi R)/2] &= (1/2)mv_{\text{top}}^2 + mg(2R) \\
 .5(1800 \text{ kg})v_1^2 + 0 - (27 \text{ nts})(232.8 \text{ m}) &= .5(1800 \text{ kg})v_{\text{top}}^2 + (1800)(9.8 \text{ m/s}^2)[2(20 \text{ m})] \\
 \Rightarrow v_1^2 &= v_{\text{top}}^2 + 791.
 \end{aligned}$$

Using N.S.L., we get:

$$\begin{aligned}
 -N - mg &= -ma_c \\
 &= -m(v_{\text{top}}^2/R).
 \end{aligned}$$

When the velocity is correct, N goes to zero and:

$$v_{\text{top}}^2 = gR.$$

Substituting that into our expression, we get:

$$\begin{aligned}
 v_1^2 &= v_{\text{top}}^2 + 791 \\
 &= gR + 791 \\
 &= (9.8 \text{ m/s}^2)(20 \text{ m}) + 791 \\
 \Rightarrow v_1 &= 31.42 \text{ m/s}.
 \end{aligned}$$

6.10)

a.) Assuming the spring is depressed a distance x , let's define the *gravitational potential energy equals zero* level to be at that point (i.e., where the spring is depressed a distance x). As such, the crate travels a distance $d + x$, where d is the crate's initial distance from the bumper. The *modified conservation of energy theorem* implies:

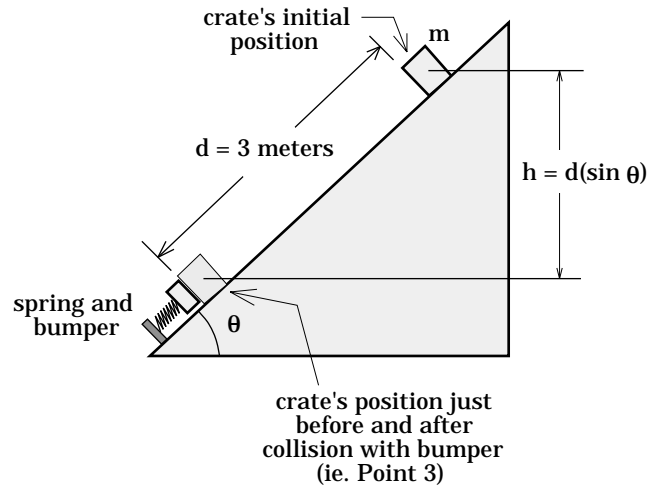
$$\begin{aligned}
 KE_1 + \Sigma U_1 + \Sigma W_{\text{ext}} &= KE_2 + \Sigma U_2 \\
 (0) + [U_{\text{grav},1} + U_{\text{sp},1}] + \Sigma W_{\text{ext}} &= (0) + [U_{\text{grav},2} + U_{\text{sp},2}] \\
 (0) + [mg(d+x) \sin \theta + (0)] - f_k(d+x) &= (0) + [(0) + .5kx^2] \\
 \Rightarrow mg(d+x) \sin \theta - f_k(d+x) &= .5kx^2 \\
 \Rightarrow (60 \text{ kg})(9.8 \text{ m/s}^2)(3 + x) \sin 55^\circ - (100 \text{ nt})(3 + x) &= .5(20,000)x^2.
 \end{aligned}$$

Rearranging:

$$10,000x^2 - 381.7x - 1145 = 0.$$

Using the Quadratic Formula, we get $x = .36$ meters.

b.) In this section, we really are not interested in x --we want to know how much energy the block has just before hitting the spring, and how much energy the block loses by the time it leaves the spring. In other words, this is really a brand new problem. As such, let's redefine the *zero gravitational potential energy level* to be at the point when the block is just about to come into contact with the spring. If that be the case, the *conservation of energy* allows us to determine the energy of the block just before striking the spring at *Point 3*:



$$\begin{aligned} KE_1 + \Sigma U_1 + \Sigma W_{\text{ext}} &= KE_3 + \Sigma U_3 \\ (0) + mg d \sin \theta - f_k d &= KE_3 + (0) \end{aligned}$$

$$\begin{aligned} \Rightarrow KE_3 &= mgd \sin \theta - f_k d \\ &= (60 \text{ kg})(9.8 \text{ m/s}^2)(3 \sin 55^\circ) - (100 \text{ nt})(3 \text{ m}) \\ &= 1144.98 \text{ joules.} \end{aligned}$$

If $3/4$ of the kinetic energy is lost, then $1/4$ is left. That means the block has kinetic energy $(1/4)(1144.98 \text{ j}) = 286.2 \text{ joules}$ as it starts back up the incline. If we let L be the distance the block travels up the incline to rest, and if we remember that the $U = 0$ level is at the spring's end, we have:

$$\begin{aligned} KE_4 + \Sigma U_4 + \Sigma W_{\text{ext}} &= KE_5 + \Sigma U_5 \\ (286.2 \text{ j}) + (0) - (100 \text{ nt})L &= (0) + (60 \text{ kg})(9.8 \text{ m/s}^2)(L \sin 55^\circ) \\ \Rightarrow L &= .49 \text{ meters.} \end{aligned}$$

6.11) Using the potential energy function provided, *conservation of energy* implies:

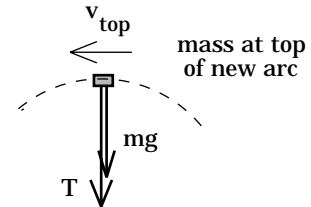
$$\begin{aligned} KE_1 + \Sigma U_1 + \Sigma W_{\text{ext}} &= KE_2 + \Sigma U_2 \\ (1/2)m_s v_1^2 + [-Gm_e m_s / (r_e + d_1)] + (0) &= (1/2)m_s v_2^2 + [-Gm_e m_s / (r_e + d_2)]. \end{aligned}$$

Noticing that the m_s 's cancel out, we can write:

$$\begin{aligned} .5(1500 \text{ m/s})^2 + [-(6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24} \text{ kg}) / (6.37 \times 10^6 \text{ m} + 1.2 \times 10^5 \text{ m})] &= \\ .5v_2^2 + [-(6.67 \times 10^{-11} \text{ m}^3/\text{kg}\cdot\text{s}^2)(5.98 \times 10^{24} \text{ kg}) / (6.37 \times 10^6 \text{ m} + .9 \times 10^5 \text{ m})]. \end{aligned}$$

SOLVING yields $v_2 = 1680 \text{ m/s}$.

6.12) The trick to this problem is in recognizing the fact that at the top of its arc, the mass is executing *centripetal acceleration* where *tension* is acting as one of the *centripetal forces* in the system. Using N.S.L. and remembering that the radius of the mass's motion is $L/3$, we get:



$$\underline{\Sigma F_c}:$$

$$\begin{aligned} -T - mg &= -ma_c \\ &= -m(v_{\text{top}}^2/R) \\ &= -m(v_{\text{top}}^2/(L/3)) \\ \Rightarrow T &= (3m/L)(v_{\text{top}})^2 - mg. \end{aligned}$$

To solve this expression for T , we need the velocity of the mass at the top of its flight.

Enter the *modified conservation of energy* approach--an approach designed specifically to determine velocities when conservative force fields are doing work on bodies. If we take the *potential energy equal to ZERO* point to be at the bottom of the arc, the information given in the sketch on the next page tells it all.

